IB Mathematics HL II
Summer Review

*Due the second week of school*

Name:______________________________

Future IB Math HL 2 students:

To enhance your chances for success in IB Mathematics HL 2, it is important that you refresh some of your Algebra II and IB Math HL 1 skills. Please do all of your work on separate sheets of paper.

I will count the completion of this packet as a homework grade for the first quarter. Please make every attempt to have the packet completed by the due date. We’ll be reviewing it together the first couple of days of class.

If you have any questions, you can contact me, Ms. Ihle, at paihle@fcps.edu
Topic 1: Algebra

Non-calculator: #1 – 10

1. Find the sum of the arithmetic series
   \[2 + 5 + 8 + 11 + \ldots + 35\]

2. Find the sum of the infinite geometric series
   \[\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} \ldots\]

3. Solve the equation \[9^{x-1} = \left(\frac{1}{3}\right)^{2x}\]

4. If \(\log_a 2 = x\) and \(\log_a 5 = y\), find in terms of \(x\) and \(y\), expressions for
   (a) \(\log_a 5\)
   (b) \(\log_a 20\)

5. Solve the equation \(\log_3 x - \log_3 (x - 5) = 1\)

6. Given that \(4 \ln 2 - 3 \ln 4 = -\ln k\), find the value of \(k\)

7. Solve the equation \(2^{2x+2} - 10 \cdot 2^x + 4 = 0\), \(x \in \mathbb{R}\)

8. Solve the equation \(e^{2-2x} = 2e^{-x}\) giving the answer as a logarithm

9. Given the \(2 + i\) is a root of the equation \(x^3 - 6x^2 + 13x - 10 = 0\), find the other two roots

10. Find \(b\) where \(\frac{2 + bi}{1 - bi} = -\frac{7}{10} + \frac{9}{10}i\)

Calculator: #11 – 13

11. A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.
   (a) Calculate the number of seats in the 20th row.
   (b) Calculate the total number of seats.

12. Find the coefficient of \(a^3b^4\) in the expansion of \((5a + b)^7\)
13. Consider the expansion of \( \left( 3x^2 - \frac{1}{x} \right)^9 \)

(a) How many terms are there in this expansion?

(b) Find the constant term in the expansion.

**Topic 2: Functions and Equations – Non-Calculator: #1 – 9**

1. Consider the function \( f(x) = 2x^2 - 8x + 5 \)

(a) Express \( f(x) \) in the form \( a(x - p)^2 + q \), where \( a, p, q \in \mathbb{Z} \)

(b) Find the minimum value of \( f(x) \)

2. The functions \( f \) and \( g \) are defined by \( f(x) = 3x \), \( g(x) = x + 2 \)

(a) Find an expression for \( (f \circ g)(x) \)

(b) Show that \( f^{-1}(18) + g^{-1}(18) = 22 \)

3. The functions \( f \) and \( g \) are defined by \( f(x) = e^x \), \( g(x) = x + 2 \)

(a) Calculate \( f^{-1}(3) \times g^{-1}(3) \)

(b) Calculate \( (f \circ g)^{-1}(3) \)

4. The function \( f \) is given by \( f(x) = e^{(x-1)^2} - 8 \)

(a) Find \( f^{-1}(x) \)

(b) Write down the domain of \( f^{-1}(x) \)

** Describe the transformations from \( f(x) \) to \( g(x) \)

5. \( f(x) = \sqrt{x} \)
   \( g(x) = 3\sqrt{x} \)

6. \( f(x) = \sin x \)
   \( g(x) = -\sin 2x \)

7. \( f(x) = \frac{1}{\sqrt{x}} \)
   \( g(x) = \frac{1}{\sqrt{-x}} \)

8. \( f(x) = x^2 \)
   \( g(x) = (x + 3)^2 - 5 \)
The diagram shows the graph of \( y = f(x) \), with the x-axis as an asymptote.

(a) On the same axes, draw the graph of \( y = f(x + 2) - 3 \), indicating the coordinates of the images of the points A and B.

(b) Write down the equation of the asymptote to the graph of \( y = f(x + 2) - 3 \)

**Topic 3: Circular Functions and Trigonometry - Non-Calculator: #0 – 13**

0. Memorize the unit circle.

1. Find the central angle measure (in radians) of an arc length 8 cm on a circle with a radius of 3 cm.

**Simplify the expressions**

2. \((\sin \theta - \cos \theta)^2 + 2 \sin \theta \cos \theta - 1\)

3. \(\sin^2 \theta + \cot^2 \theta \sin^2 \theta\)

4. \(\sin^2 85^\circ + \sin^2 5^\circ\)

5. If \( \csc u = \frac{13}{5} \) and \( \frac{\pi}{2} < u < \pi \), find the exact value of \( \sin 2u \), \( \cos 2u \), and \( \tan 2u \).
**Find all solutions of the equation in the interval \([0, 2\pi]\)**

6. \(\sin 2x + \cos x = 0\)  
7. \(\sin 2x \cos x - \cos 2x \sin x = 0\)  
8. \(\sin x - \sqrt{3} \cos x = 0\)  
9. \(\sin^2 x + \cos 2x = 1\)

10. The graph of a function of the form \(y = p \cos qx\) is given in the diagram below.

(a) Write down the value of \(p\).
(b) Calculate the value of \(q\).

11. Given that \(\tan 2\theta = \frac{3}{4}\), find the possible values of \(\tan \theta\).

12. The angle \(\theta\) satisfies the equation \(2 \tan^2 \theta - 5 \sec \theta - 10 = 0\), where \(\theta\) is in the second quadrant. Find the value of \(\sec \theta\).

13. The lengths of the sides of a triangle ABC are \(x - 2\), \(x\), and \(x + 2\). The largest angle is 120°.

(a) Find the value of \(x\).

(b) Show that the area of the triangle is \(\frac{15\sqrt{3}}{4}\).

(c) Find \(\sin A + \sin B + \sin C\) giving your answer in the form \(\frac{p\sqrt{q}}{r}\), where \(p, q, r \in \mathbb{Z}\).
14. The following diagram shows a circle of centre O, and radius 15 cm. The arc ACB subtends an angle of 2 radians at the centre O.

(a) Find the length of the arc ACB.

(b) Find the area of the shaded region.

15. The following diagram shows a triangle with sides 5 cm, 7 cm, and 8 cm.

(a) Find the size of the smallest angle, in degrees.

(b) Find the area of the triangle.
16. A farmer owns a triangular field ABC. One side of the triangle, [AC], is 104 m, a second side, [AB], is 65 m and the angle between these two sides is 60°.

(a) Use the cosine rule to calculate the length of the third side of the field.

(b) Given that \( \sin 60° = \frac{\sqrt{3}}{2} \), find the area of the field in the form \( p\sqrt{3} \) where \( p \) is an integer.

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts \( A_1 \) and \( A_2 \) by constructing a straight fence [AD] of length \( x \) metres, as shown on the diagram below.

(c) (i) Show that the area of \( A_1 \) is given by \( \frac{65x}{4} \).

(ii) Find a similar expression for the area of \( A_2 \).

(iii) Hence, find the value of \( x \) in the form \( q\sqrt{3} \), where \( q \) is an integer.

(d) (i) Explain why \( \sin \hat{ADC} = \sin \hat{ADB} \).

(ii) Use the result of part (i) and the sine rule to show that \( \frac{BD}{DC} = \frac{5}{8} \).
Topic 5: Statistics and Probability (Review from Algebra 2)

1. Use the given data to answer the following:
   3, 3, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 7, 7, 8, 8
   (a) Find the minimum, maximum, and range
   (b) Find the median
   (c) Find the first quartile and third quartile
   (d) Graph the box and whisker plot
   (e) Find the mode
   (f) Find the arithmetic mean (average)
   (g) Find the variance and standard deviation

2. Use the given data to answer the following:
   1, 1, 2, 3, 4, 5, 5, 5, 6, 7, 7, 8, 8, 9, 10, 17, 19, 20
   (a) Find the minimum, maximum, and range
   (b) Find the median
   (c) Find the first quartile and third quartile
   (d) Graph the box and whisker plot
   (e) Find the mode
   (f) Find the arithmetic mean (average)
   (g) Find the variance and standard deviation

3. A normal distribution has a mean of 80 and a standard deviation of 5. Convert the following numbers into $z$-scores:
   (a) 80
   (b) 85
   (c) 65
   (d) 92.5
4. Use the z-table or a graphing calculator to find the following probabilities:
   (a) \( P(z < 1.25) \)
   (b) \( P(z > 0.32) \)
   (c) \( P(-1.96 < z < 1.96) \)

5. A company produces light bulbs having a life expectancy that is normally distributed with a mean of 1,000 hours and a standard deviation of 50 hours.
   (a) What percent of bulbs will last for 1,000 hours or more?
   (b) What is the probability that a randomly chosen bulb will burn out in 900 hours of less?
   (c) What is the probability that a randomly chosen bulb will last between 920 hours and 1,125 hours?

6. A normally distributed set of 997 values has a mean of 100 and a standard deviation of 10. What is the closest to the number of values expected to be above 120?

7. Find the number of ways that 3 out of 10 people can be arranged on a stage.

8. In how many ways can a president and vice-president be chosen out of seven candidates?

9. If four friends race to the cafeteria line, how many ways can they end up in line?

10. How many 12 member juries can be selected from a field of 15 people?

11. Student leadership has 16 elected positions. Six of them are boys and ten of them are girls. They need to make a 5 person committee. How many ways can the committee have 3 boys?

**Topic 6: Calculus - Non-Calculator**

1. Evaluate the limit
   (a) \( \lim_{x \to 2} \left( \frac{x^2 - 3x + 2}{x - 2} \right) \)
   (b) \( \lim_{x \to 4} \left( \frac{x - 1}{\sqrt{x - 1}} \right) \)
   (c) \( \lim_{x \to 1} \left( \frac{1 - x}{x - 1} \right) \)
   (d) \( \lim_{x \to -4} \left( \frac{x^3 + 64}{x + 4} \right) \)
   (e) \( \lim_{x \to \infty} \left( \frac{2x^3 - 6x^2 + 3x - 1}{5x^3 - 8x + 9} \right) \)
   (f) \( \lim_{x \to \infty} \left( \frac{2x^3 - 6x^2 + 13x - 2}{5x^4 - 8x^2 + 9} \right) \)
   (g) \( \lim_{x \to \infty} \left( \frac{5x^3 - 7x^2 + 3x - 2}{-8x^2 + 9x - 3} \right) \)
   (h) \( \lim_{x \to 0} \left( \frac{1}{x - 7} \right) \)
2. Find the value of $k$ that makes the piecewise function $f(x)$ continuous.

$$f(x) = \begin{cases} 
  kx^2 - 1 & \text{for } x > -1 \\
  -2x + 3 & \text{for } x \leq -1
\end{cases}$$

3. If $f(x) = 3x^2 - 2x + 5$, use the definition of the derivative to find the equation of the tangent line at $x = -4$.

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

4. Find the derivative of each function:

(a) $f(x) = \frac{4}{3} \pi x^3$

(b) $f(x) = \ln e \cdot x^3$

(c) $f(x) = 3x^e$

(d) $f(x) = a^{\log_a x}$

(e) $f(x) = \sin^2 x + \cos^2 x$

(f) $f(x) = x^x$

(g) $f(x) = 3x(2x + 5)^4$

(h) $f(x) = \frac{2x^3 - 5x}{3x + 7}$