All students entering AP Calculus AB are expected to be proficient in Pre-Calculus skills. To enhance your chances for success in this class, it is important that you refresh these topics. You need to show your work on separate sheets of paper. I will count the completion of this packet as 5 homework grades for the first quarter. The material covered in this assignment will be tested during the 1st Quarter.

Please send me an e-mail at Hoon.Kim@fcps.edu if you have any question.
**Topic 1 Equation of a line**

1. Determine the slope of the line that passes through the points (-1, 6) and (11, -6).

2. Find the equation of the line that passes through the point (1, -1) and has a slope of -3.
   Leave your answer in point-slope form.

3. Find an equation of the line that passes through (-1, -3) parallel to the line 
   \( 2x + y = 19 \).
   Leave your answer in slope-intercept form.

4. Find an equation of the line that passes through (8, 17) and is perpendicular to the line 
   \( x + 2y = 2 \).
   Leave your answer in standard form.

**Topic 2 Functions**

**A. Composition of functions**

**Find two functions** \( f \) and \( g \) such that \( h(x) = [f \circ g](x) \). Neither function may be the identity function \( y = x \).

1. \( h(x) = \sqrt{x^2 - 4} \)
2. \( h(x) = \frac{1}{x^2 - 6x + 9} \)
3. \( h(x) = (x + 3)^2 + 5(x + 3) + 7 \)

**Evaluate each expression using the values in the table.**

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

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<th>( x )</th>
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<th>1</th>
<th>4</th>
<th>9</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**If** \( f(x) = \frac{1}{x} \), \( g(x) = x^2 \), and \( h(x) = \sqrt{x - 3} \), find the composition of two functions and state the domain.

1. \( g[f(x)] \)
2. \( g[h(x)] \)
3. \( f[h(x)] \)
4. \( f[f(x)] \)

**B. Inverse Functions**

1. If \( f(x) = 3x^3 - 1 \), find its inverse, \( f^{-1}(x) \).

2. Show that \( f(x) = e^{x^3} + 2 \) and \( g(x) = \ln(x - 2) + 3 \) are inverses each other.

**C. Even and Odd Functions**

**Determine if the following functions are even, odd, or neither.**

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>-2x^3 + 3x^2 - 7x</th>
<th>3x^3 - 2x^2 + 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>5x^3 + x + 2</td>
<td>( e^x - e^{-x} )</td>
</tr>
<tr>
<td>( f(x) )</td>
<td>( \frac{x^2}{x^4 + 5} )</td>
<td>( \frac{-x}{x + 1} )</td>
</tr>
</tbody>
</table>
**Topic 3 Factor**

**Factor completely.**

1. \(x^4 - 81\)
2. \(54x^3 + 250y^3\)
3. \(3x^2 - 36xy + 108y^2\)
4. \(x^2 + 14x + 49 - 81y^2\)
5. \(x^3 - xy^2 + x^2y - y^3\)
6. \((x - 3)^2 (2x+1)^3 + (x - 3)^3 (2x+1)^2\)

**Topic 4 Solving Polynomial and Rational Equations**

1. \(7x^2 - 5x = 0\)
2. \(x^3 - 4x^2 + x + 6 = 0\)
3. \((3x - 1)^2 = 32\)
4. \(3x^3 - 24x^2 + 21x = 0\)
5. \(x^2 - 6x + 1 = 0\)
6. \(3x^2 - 6x + 2 = 0\)
7. \(\frac{1}{x-3} - \frac{2}{x+3} = \frac{2x}{x^2 - 9}\)
8. \(x + \frac{1}{x} = \frac{13}{6}\)

**Topic 5 Exponents & Logarithms**

**Simplify the expression.**

1. \(\log_8 \frac{1}{16}\)
2. \(e^{2\ln 5}\)
3. \(\frac{\ln 8}{\ln 2}\)

**Expand the expression using the property of logarithms.**

1. \(\log \left[ \frac{\sqrt[3]{y}}{x^2 z^5} \right]\)
2. \(\ln \left[ \frac{5x}{\sqrt{x - 7 (3x + 5)}} \right]\)

**Condense the expression using the property of logarithms.**

1. \(\frac{1}{2} \log (x + 5) - 2 \log x + 3 \log (x - 2) - 5 \log (x + 1)\)
2. \(2 \left[ \ln (x - 1) - 3 \ln (x + 2) - \frac{1}{3} \ln (x + 5) \right]\)

**Solve the equation.**

1. \(\log_8 (x - 5) = \frac{2}{3}\)
2. \(\log (5x) + \log (x - 1) = 2\)
3. \(4^{3x} = 8^{x+1}\)
4. \(5^x = 3e^x\)
5. \(2e^{-x} - 3 = 11\)
6. \(3^{5x+1} = 5^{2x-3}\)
**Describe the transformations from \( f(x) \) to \( g(x) \), where \( g(x) \) is defined below.**

1. \( g(x) = f\left(\frac{x}{5}\right) \)

2. \( g(x) = \frac{1}{7} f(x) \)

3. \( g(x) = -f(x) \)

4. \( g(x) = f(-x) \)

5. \( g(x) = 9f(x) \)

6. \( g(x) = f(x-3) + 5 \)

**Sketch the following graph using \( f(x) = x^2 - 4x + 3 \)**

1. \( y = f(x+2) - 3 \)

2. \( y = f(-x) \)

3. \( y = |f(x)| \)

4. \( y = f(|x|) \)
Topic 7 Function Analysis

1. Use the complete graph of a polynomial function \( f(x) \) to answer the following: \( x \in [-4, 10] \) and x-scale is 1

A. Is the degree of \( f(x) \) even or odd?

B. Is the leading coefficient of \( f(x) \) positive or negative?

C. What are the distinct real zeros of \( f(x) \)?

D. What is the least degree of \( f(x) \)?

E. How many turning points does it have?

F. Stationary inflection point occurs at what value of \( x \)?

G. Describe the end behaviors

2. Use the graph of \( f(x) = -\frac{5x}{x^2 - 4} \) to answer the following:

A. Find the domain

B. Find the equation of vertical asymptote

C. Find the equation of the horizontal asymptote

D. Find the range

E. Is it an even function or odd function?

F. Describe the vertical asymptotic behaviors
   \[ \lim_{x \to -2} f(x) = \quad \lim_{x \to 2} f(x) = \]
3. Use the graph of \( f(x) = \frac{16x}{x^3 - 4x} \) to answer the following:

A. Find the domain.

B. Find the equation of the vertical asymptote.

C. Find the equation of the horizontal asymptote.

D. Find the range.

E. Is it an even function or odd function?

F. Describe the vertical asymptotic behaviors

\[
\lim_{x \to 2^-} f(x) = \quad \lim_{x \to 2^+} f(x) =
\]

4. Use the graph of \( f(x) = \frac{x^2 - 3x - 4}{x - 2} \) to answer the following:

A. Find the domain.

B. Find the equation of the vertical asymptote.

C. Find the x-intercepts and y-intercept.

D. Find the equation of the slant asymptote.

E. Find the range.
5. Graph and analyze $f(x) = -2 \cdot e^{-x} + 4$

A. Parent function:
B. Domain:
C. Horizontal Asymptote:
D. Describe the behavior near the horizontal asymptote using the limit notation.
E. Range:
F. Describe the behavior of the function using the concavity.
G. Key points:
H. Graph the function.

**Topic 8 Piecewise Functions**

**Use $f(x) = \begin{cases} 
-x, & x \leq 3 \\
\frac{2}{3}x - 4, & x > 3 
\end{cases}$ to answer the following:**

1. Evaluate the function.
   A. $f(-3)$
   B. $f(0)$
   C. $f(3)$
   D. $f(6)$

2. Graph the function.

3. Is the function continuous at $x = 3$?
** Use \( f(x) = \begin{cases} -2x - 6, & x < -1 \\ 2x - 2, & x \geq -1 \end{cases} \) to answer the following:

1. Evaluate the function.
   A. \( f(-3) \)
   B. \( f(-1) \)
   C. \( f(0) \)
   D. \( f(5) \)

2. Graph the function.

3. Is the function continuous at \( x = -1 \)？

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** Topic 9 Trigonometry **

The following Trigonometric Identities MUST be memorized.

<table>
<thead>
<tr>
<th>Reciprocal Identities</th>
<th>Quotient Identities</th>
<th>Pythagorean Identities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin x = \frac{1}{\csc x} )</td>
<td>( \tan x = \frac{\sin x}{\cos x} )</td>
<td>( \sin^2 x + \cos^2 x = 1 )</td>
</tr>
<tr>
<td>( \csc x = \frac{1}{\sin x} )</td>
<td>( \cot x = \frac{\cos x}{\sin x} )</td>
<td>1 + ( \tan^2 x = \sec^2 x )</td>
</tr>
<tr>
<td>( \cos x = \frac{1}{\sec x} )</td>
<td></td>
<td>( 1 + \cot^2 x = \csc^2 x )</td>
</tr>
<tr>
<td>( \sec x = \frac{1}{\cos x} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tan x = \frac{1}{\cot x} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \cot x = \frac{1}{\tan x} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Co-Function Identities</th>
<th>Odd / Even Identities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \left( \frac{\pi}{2} - \theta \right) = \cos \theta )</td>
<td>Odd</td>
</tr>
<tr>
<td>( \cos \left( \frac{\pi}{2} - \theta \right) = \sin \theta )</td>
<td>Even</td>
</tr>
<tr>
<td>( \csc \left( \frac{\pi}{2} - \theta \right) = \sec \theta )</td>
<td></td>
</tr>
<tr>
<td>( \sec \left( \frac{\pi}{2} - \theta \right) = \csc \theta )</td>
<td></td>
</tr>
<tr>
<td>( \tan \left( \frac{\pi}{2} - \theta \right) = \cot \theta )</td>
<td></td>
</tr>
<tr>
<td>( \cot \left( \frac{\pi}{2} - \theta \right) = \tan \theta )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Double-Angle Identities</th>
<th>Power-Reducing Identities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin(2x) = 2\sin x \cdot \cos x )</td>
<td>( \sin^2 x = \frac{1 - \cos 2x}{2} )</td>
</tr>
<tr>
<td>( \cos(2x) = \cos^2 x - \sin^2 x )</td>
<td>( \cos^2 x = \frac{1 + \cos 2x}{2} )</td>
</tr>
<tr>
<td>( = 1 - 2\sin^2 x )</td>
<td></td>
</tr>
<tr>
<td>( = 2\cos^2 x - 1 )</td>
<td></td>
</tr>
</tbody>
</table>
The Radian Measures and Coordinates Must be memorized.

\[
\sin \theta = \frac{y}{r} \quad \csc \theta = \frac{r}{y}
\]

\[
\cos \theta = \frac{x}{r} \quad \sec \theta = \frac{r}{x}
\]

\[
\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}
\]

In a unit circle, \( x = \cos \theta \) and \( y = \sin \theta \)

** Evaluate each expression.**

1. \( \arcsin \left( -\frac{1}{2} \right) \)

2. \( \sin^{-1} \left( -\frac{\sqrt{2}}{2} \right) \)

3. \( \arccos \left( -\frac{\sqrt{3}}{2} \right) \)

4. \( \cos^{-1} (-1) \)

5. \( \arctan (-\sqrt{3}) \)

6. \( \tan^{-1} (0) \)

7. \( \cos \left( -\frac{3\pi}{4} \right) \)

8. \( \sin \left( -\frac{9\pi}{4} \right) \)

9. \( \tan \left( \frac{5\pi}{6} \right) \)

10. \( \csc \left( \frac{3\pi}{2} \right) \)

11. \( \cot ( -\pi ) \)

12. \( \sec \left( \frac{\pi}{3} \right) \)
**Simplify the expressions.**

1. \( \sin^2 \theta + \cot^2 \theta \cdot \sin^2 \theta \)
2. \( \sin^2 \left( \frac{\pi}{6} \right) + \cos^2 \left( \frac{\pi}{3} \right) \)

**Use \( \cos(2\theta) = \cos^2 \theta - \sin^2 \theta = 1 - 2\sin^2 \theta = 2\cos^2 \theta - 1 \) to derive the Power-Reducing Identities.**

1. \( \sin^2 x = \frac{1 - \cos 2x}{2} \)
2. \( \cos^2 x = \frac{1 + \cos 2x}{2} \)

**Find all solutions of the equation in the interval \([0, 2\pi)\)**

1. \( 2 \cos x - \sqrt{3} = 0 \)
2. \( \sec^2 x = \sec x + 2 \)

3. \( 2 \sin x \cdot \cos x + \cos x = 0 \)
4. \( \sin x - \sqrt{3} \cos x = 0 \)

**Topic 10 Limits**

**Evaluate Limits Graphically.**

1. Use the graph of \( f(x) \) to answer the following:

   A. Evaluate \( \lim_{x \to 2} f(x) \)
   
   B. Evaluate \( \lim_{x \to 2} f(x) \)
   
   C. Evaluate \( \lim_{x \to 2} f(x) \)
   
   D. Evaluate \( f(2) \)
   
   E. Is \( f(x) \) continuous at \( x = 2 \)?
2. Use the graph of \( f(x) \) to answer the following:

A. Evaluate \( \lim_{{x \to 3}} f(x) \)

B. Evaluate \( \lim_{{x \to 3^+}} f(x) \)

C. Evaluate \( \lim_{{x \to 3^-}} f(x) \)

D. Evaluate \( f(3) \)

E. Is \( f(x) \) continuous at \( x = 3 \) ?

** Evaluate Limits Algebraically.

1. \( \lim_{{x \to -3}} (-2x^2 + 1) \)

2. \( \lim_{{x \to 3}} \dfrac{x^2 - 5x}{x^2 - 25} \)

3. \( \lim_{{x \to 3}} \dfrac{x^2 - x - 6}{x - 3} \)

4. \( \lim_{{x \to 3}} \dfrac{x - 4}{\sqrt{x} - 2} \)

5. \( \lim_{{x \to -2}} \dfrac{1 - 2}{x} \)

6. \( \lim_{{x \to 3}} \dfrac{x^3 - 27}{x - 3} \)

7. \( \lim_{{x \to -3}} \dfrac{x^2 + 2}{2x^2 + 5} \)

8. \( \lim_{{x \to 5}} \dfrac{2x^2 + 3x}{5x^3 - 2x + 1} \)

9. \( \lim_{{x \to -3}} \dfrac{1 - x^2}{1 + x} \)

10. \( \lim_{{x \to -3}} \dfrac{3000x^3}{x - 1000x^3} \)
**Continuity**

1. Find the value of $k$ that makes the piecewise function $f(x)$ be continuous.

   
   
   
   
   $\begin{align*}
   f(x) &= \begin{cases} 
   kx^2 - 1 & \text{for } x > -1 \\
   -2x + 3 & \text{for } x \leq -1
   \end{cases}
   \end{align*}$

2. Write down the piecewise function so that $f(x) = \frac{\sin x}{x}$ is continuous for all real numbers.

**Topic 11 Derivatives**

**Definition of the Derivative function**

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

1. Find the equation of the tangent line to the graph of $f(x) = 3x^2 - 2x + 5$ at $x = -1$ using the definition of the derivative.

2. Evaluate $\lim_{h \to 0} \frac{[2(3+h)^2 - 7(3+h) + 9] - [2(3)^2 - 7 \cdot 3 + 9]}{h}$

**Power Rule**

1. Find the equation of the normal line to the graph of $f(x) = \frac{7}{x}$ at $(1, 7)$ using the power rule.

**Find the derivative of each function using the power rule.**

1. $f(x) = \sqrt[3]{x^2}$

2. $f(x) = \frac{1}{\sqrt[3]{x}}$

3. $f(x) = \frac{1}{x^2}$

4. $f(x) = 3x^e$

5. $f(x) = \frac{4}{3} \pi x^3$

6. $f(x) = \ln e \cdot x^3$